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M5-brane geometries, T-duality and fluxes

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Abstract: We describe a duality relation between configurations of M5-branes in M-theory and type IIB theory on Taub-NUT geometries with NSNS and RR 3-form field strength fluxes. The flux parameters are controlled by the angles between the M5-brane and the (T)duality directions. For one M5-brane, the duality leads to a family of supersymmetric flux configurations which interpolates between imaginary self-dual fluxes and fluxes similar to the Polchinski-Strassler kind. For multiple M5-branes, the IIB configurations are related to fluxes for twisted sector fields in orbifolds. The dual M5-brane picture also provides a geometric interpretation for several properties of flux configurations (like the supersymmetry conditions, their contribution to tadpoles, etc), and for many non-trivial effects in the IIB side. Among the latter, the dielectric effect for probe D3-branes is dual to the recombination of probe M5-branes with background ones; also, a picture of a decay channel for non-supersymmetric fluxes is suggested.

Keywords: M-Theory, D-branes, String Duality.

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Contents

1. Introduction

String and M-theory backgrounds with field strength fluxes for various p-form gauge fields have been shown to lead to interesting features [1]–[5]. For instance, scalar potentials with non-trivial minima leading to moduli stabilization, supergravity descriptions in terms of warped geometries, and (possibly partial) breaking of supersymmetry. These configurations are presently under intense study from diverse points of view. In this paper we would like to present a new approach to their analysis in terms of a new dual picture.

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We describe a duality between configurations of M-theory M5-branes and type IIB string theory on Taub-NUT geometries with a background of NSNS and RR 3-form field strength fluxes. Data of the flux configuration, and many of its properties, are encoded in simple properties of the geometry of the dual M5-brane configuration.

The duality is a generalization of the relation between parallel M5-branes in M-theory and Taub-NUT geometries in IIB theory, which we review in section 2. In section 3 we present our duality and show that M5-branes at angles with respect to the duality directions are related to Taub-NUT geometries with a non-trivial background of 3-form fluxes, controlled by the dual angle parameters (sections 3.1.1 and 3.1.2).

These configurations are interesting for several reasons. Different properties of the IIB backgrounds are encoded in the M5-brane geometry, for instance flux quantization conditions (section 3.1.3), and charges induced by the flux background (section 3.1.4). Moreover, in section 3.1.5 we argue that the IIB duals to single stacks of M5-branes preserve 16 supersymmetries, and provide a family of backgrounds that interpolate between imaginary self-dual fluxes with constant dilaton and more general supersymmetric flux configurations with varying dilaton, reminiscent of those in [6].

In section 3.2 we discuss multiple M5-brane stacks, for which the M-theory dual provides a description of twisted fluxes on $\mathbb{C}^2/\mathbb{Z}_N$ orbifold singularities. Again some features like moduli stabilization or the supersymmetry of the configuration are explained using the M5-brane geometry (section 3.2.2). The M-theory picture can be also used to suggest decay mechanisms for non-supersymmetric fluxes (section 3.2.3), and the stabilization by fluxes of unstable non-Calabi-Yau geometries (section 3.2.4).

In section 3.3 we discuss the M-theory picture of diverse effects when one introduces D3-brane probes in the IIB flux background, for instance, the appearance of 4d $\mathcal{N}=2$ soft masses (section 3.3.1), and Myers dielectric effect [7] (section 3.3.2). A prominent role is played by the process of recombination of intersecting M5-branes. Finally, in section 4 we discuss generalizations of the above duality to threefold geometries related to the conifold. Section 5 contains our final comments.

2. M5-brane vs. Taub-NUT

In this section we review the geometry of Taub-NUT and its duality relation with M-theory M5-branes.

2.1 A few facts on Taub-NUT

The N-center Taub-NUT metric is (see e.g. [8] for a useful reference)

$$\begin{aligned} ds^2 &= H(\vec{x})^{-1}(dy+\omega_i dx^i)^2 + H(\vec{x})d\vec{x}^{\,2} \\ H(\vec{x}) &= 1+\sum_{a=1}^{N} H_a(\vec{x})\,; \qquad H_a(\vec{x}) = \frac{1}{|\vec{x}-\vec{x}_a|} \\ d\omega &= *_{3d}\, dH(\vec{x})\,, \end{aligned} \tag{2.1}$$

where i runs over three indices and $\vec{x}=(x^i)$. Also $\omega=\omega_i dx^i$ and $*_{3d}$ denotes Hodge duality in the $\mathbb{R}^3$ parametrized by $\vec{x}$.

The geometry corresponds to an $\mathbf{S^1}$ fibration over a base $\mathbb{R}^3$ (parametrized by $\vec{x}$). The fiber has constant asymptotic radius (fixed to unity in the above expression) and degenerates to zero radius over the locations $\vec{x}_a$, known as centers of the geometry. The $\mathbf{S^1}$ bundle over a 2-sphere $(\mathbf{S^2})_\mathbf{k}$ in $\mathbb{R}^3$ surrounding k of these centers has Chern class k, namely

$$\int_{(\mathbf{S^2})_\mathbf{k}} d\omega = k \tag{2.2}$$

so is a Hopf bundle over a two-sphere surrounding each center.

The geometry contains $N-1$ homologically independent non-trivial compact 2-cycles. A non-trivial 2-cycle Σ_{ab} can be obtained by fibering the $\mathbf{S^1}$ fiber over the segment in $\mathbb{R}^3$ joining the centers $\vec{x}_a$ and $\vec{x}_b$. Out of these Σ_{ab}, only $N-1$ are linearly independent, a simple basis is provided by the cycles $\Sigma_{a,a+1}$, for $a = 1, \ldots, N-1$.

The geometry also contains N cohomologically independent normalizable 2-forms,

$$\Omega_a = d\chi_a\,; \qquad \chi_a = H^{-1}H_a(dy+\omega) - \omega_a\,, \tag{2.3}$$

where ω_a is defined by $d\omega_a = *_{3d} dH_a(\vec{x})$. With suitable normalization they obey the orthonormality condition $\int_{TN} \Omega_a \wedge \Omega_b = \delta_{ab}$.

The form Ω_a has support localized near the center $\vec{x}_a$. It is useful to introduce linear combinations $\Omega = \sum_{a=1}^N \Omega_a$ and $\Omega_{ab} = (\Omega_a - \Omega_b)/2$. The latter are Poincare dual to the 2-cycles Σ_{ab} (with suitable orientation). In the limit of infinite asymptotic radius, the geometry corresponds to an N-center ALE geometry, namely a (generically blown-up) $\mathbb{C}^2/\mathbb{Z}_N$ orbifold. In this limit, the form Ω becomes non-normalizable, and can be regarded as a constant 2-form inherited from the covering $\mathbb{C}^2$. The forms

$$\Omega_k = \frac{1}{N}\sum_{a=1}^{N} e^{2\pi i\, ka/N}\,\Omega_a \tag{2.4}$$

(with a understood modulo N) belong to the k^{th} twisted sector of the orbifold. We will hence denote the forms $\Omega \equiv \Omega_0$, $\Omega_{k\neq 0}$ as untwisted and twisted, respectively.

For a one-center metric, $H = 1 + 1/r$ with $r = |\vec{x}|$, and the harmonic 2-form is

$$\Omega = -d(H^{-1})(dy+\omega) - H^{-1}d\omega\,. \tag{2.5}$$

2.2 The duality relation

We begin by reviewing the duality relation between M-theory configurations of parallel M5-branes (with two transverse coordinates compactified on a two-torus) with type IIB on a Taub-NUT metric background. This relation is usually phrased as a T-duality between IIA NS5-branes and IIB on Taub-NUT geometries [9], but the former formulation makes the geometry of forthcoming configurations more transparent.

Consider a set of N M5-branes with world-volume along the directions 012345, and sitting at a point in the remaining ones, 678910. Let 6,10 be compactified on a two-torus that we momentarily take square for simplicity.

The relation to IIB on Taub-NUT is most easily established using the supergravity description of the M-theory configuration. It is convenient to use the solution for M5-branes smeared in the directions 6, 10 (see [10] for a discussion of the localized M5-brane solution and T-duality). It reads

$$ds^2_{11d} = H_5(\vec{x})^{-1/3}\, ds^2_{012345} \ + H_5(\vec{x})^{2/3}\, ds^2_{678910}\,, \tag{2.6}$$

where $H_5(\vec{x}) = 1 + \sum_a \frac{1}{|\vec{x}-\vec{x}_a|}$, and $\vec{x}_a$ denote the M5-brane locations in 789. For simplicity we consider the case of coincident centers $\vec{x}_a = 0$, for which $H_5 = 1 + \frac{N}{r}$, with $r = |\vec{x}|$.

Moreover, the M5-branes are magnetically charged under the M-theory 3-form. Hence there is a 3-form background $C_3 = \omega\, dx^6\, dx^{10}$ (wedge products are implicit throughout the paper), with $d\omega = *_{3d} dH_5$. Upon reduction to IIA in the direction 10, using the standard ansatz

$$ds^2 = e^{4\phi/3}\,(\, dx^{10} \ + \ A_M\, dx^M\,)^2 \ + \ e^{-2\phi/3}\, g^{IIA}_{MN}\, dx^M\, dx^N \tag{2.7}$$

we obtain the IIA background

$$\begin{aligned} ds^2_{IIA} &= ds^2_{012345} + H_5\, ds^2_{6789} \\ e^{2\phi} &= H_5 \\ B^{NS} &= \omega\, dx^6\,. \end{aligned} \tag{2.8}$$

Application of T-duality formulae in appendix A, leads to the type IIB background fields

$$\begin{aligned} \tau_{IIB} &= a + ie^{-\phi} = i \\ ds^2_{IIB} &= ds^2_{012345} + H_5^{-1}(r)(dx^6 + \omega_i dx^i)^2 + H_5(r) d\vec{x}^{\,2}\,. \end{aligned} \tag{2.9}$$

This corresponds to a purely metric background, corresponding to a Taub-NUT geometry (2.1) with N coincident centers. The IIB complex coupling takes the value $\tau = i$ because we took equal radii for the 6, 10 circles. Applying the dualities to the background with general radii leads to $\tau = iR_6/R_{10}$.

The general mapping of parameters between both configurations is as follows. The location of the a^{th} M5-brane in 7, 8, 9 maps to the position $\vec{x}_a$ of the a^{th} Taub-NUT center in the base $\mathbb{R}^3$. The location of the a^{th} M5-brane in 6, 10 is mapped to the component of the NSNS and RR 2-form fields along Ω_a. Namely, the positions (normalized w.r.t. the total radius, i.e. $\phi^i = x^i/R_i$) correspond to the coefficients ϕ_a^6, ϕ_a^{10} in the expansion

$$B_{NSNS} = \sum_a \phi_a^6\, \Omega_a\,; \qquad B_{RR} = \sum_a \phi_a^{10}\, \Omega_a\,. \tag{2.10}$$

See [11] for further discussion. The bottomline of this section is that simultaneous shrinking of the directions 6, 10 in the M5-brane configuration leads to IIB theory on a Taub-NUT geometry.

3. M5-brane geometries and fluxes

In this section we consider a generalization of the above duality. It is a duality between configurations of M5-branes at angles and Taub-NUT geometries with 3-form field strength fluxes. Intuitively, the amount of rotation of a given M5-brane with respect to the 4, 5 coordinates will map to the amount of NSNS and RR flux turned on along the corresponding harmonic 2-form in the dual Taub-NUT geometry. Thus, many features of fluxes are easily geometrized into configurations of M5-branes.

3.1 Single M5-brane

3.1.1 Intuitive T-duality

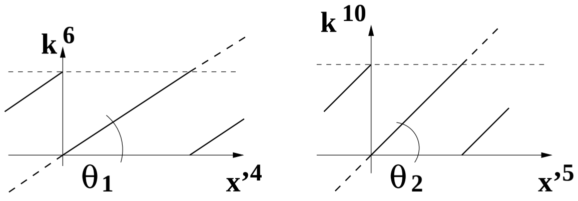

Figure 1: Configuration of M5-brane at angles (with respect to the duality directions).

Let us start by considering an intuitive explanation of the basic duality we explore in the present paper. Let us regard the M-theory spacetime as the product of a $\mathbf{M_4}$ (spanned by 0123), times $\mathbb{R} \times \mathbf{S^1}$ (spanned by 46), times $\mathbb{R} \times \mathbf{S^1}$ (spanned by 510), times an $\mathbb{R}^3$ (spanned by 789). Consider a single M5 brane with volume spanning $\mathbf{M_4}$ times a real line in each of the $\mathbb{R} \times \mathbf{S^1}$ factors. We denote by θ_1, θ_2 the angle between these lines and the directions 4 and 5, respectively. See figure 1.

Let us shrink the directions 6, 10 and obtain the corresponding dual type IIB configuration. In order to do that, it is useful to regard the original M-theory configuration as that of one M5-brane spanning 0, 1, 2, 3, 4, 5 and with a non-trivial profile for the world-volume scalars ϕ^6, ϕ^{10} that encode the location of the M5-brane in the directions 6, 10, of the form

$$\phi^6 \;=\; \tan\theta_1\, x^4\,; \qquad \phi^{10} \;=\; \tan\theta_2\, x^5\,. \tag{3.1}$$

Under the duality, we obtain the IIB theory on a Taub-NUT geometry. Recall that the scalars are mapped to the components of B_{NSNS}, B_{RR} along Ω. The linear variation of these scalars with x^4, x^5 then implies that there are non-trivial NSNS and RR 3-form field strength fluxes turned on the Taub-NUT, roughly of the form

$$H_3 \;=\; \tan\theta_1\, \Omega\, dx^4\,; \qquad F_3 \;=\; \tan\theta_2\, \Omega\, dx^5 \tag{3.2}$$

(Normalization can easily be fixed from flux quantization conditions, see section 3.1.3).

The generalization to N parallel M5-branes is straightforward, namely the IIB dual is given by an N-center Taub-NUT space, with fluxes (3.2) along the overall (untwisted) 2-form Ω.

Hence we have found a duality relation between configurations of 3-form field strength fluxes in Taub-NUT space and the geometry of the M5-brane. The above analysis is however oversimplified, e.g. it ignores the backreaction of the fluxes in the geometry. In next section we describe a more careful application of T-duality, that reproduces these effects.

3.1.2 Detailed duality relation

Let us derive the above duality relation with an argument closer to that in section 2.2. Consider the supergravity solution corresponding to M5-branes along 012345, in a background Minkowski metric, (2.6). In order to introduce the above tilting of the M5-brane,

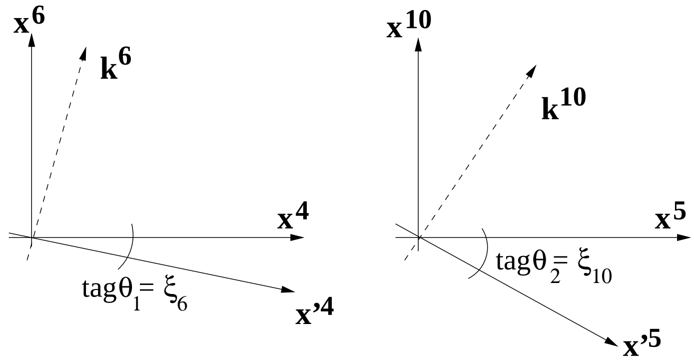

Figure 2: Pictorial depiction of the effect of changing coordinates. The directions which are shrunk in the duality are at fixed values of the new coordinates x'^4, x'^5, hence correspond to the Killing directions k_6, k_{10}. Hence we are describing a (T)duality of the M5-brane configuration along directions at angles with the latter. The angles θ_1, θ_2 are determined by the coordinate change parameters ξ_6, ξ_{10}. In order to avoid annoying minus signs in our formulas, we consider clockwise angles as positive.

let us perform the change of variables

$$x^4 = x'^4 + \xi_6 \, x^6 \, ; \qquad x^5 = x'^5 + \xi_{10} \, x^{10} \tag{3.3}$$

As shown in figure 2, this implies that the M5-brane is along directions at angles with respect to the directions along which we perform the dualities.

The metric becomes

$$\begin{aligned} ds^2_{11d} = {} & H_5^{-1/3} \, ds^2_{0123} + H_5^{2/3} \, ds^2_{789} + H_5^{-1/3}(H_5+\xi_6^2)(dx^6)^2 + H_5^{-1/3}(H_5+\xi_{10}^2)(dx^{10})^2 + \\ & + H_5^{-1/3}(dx'^4)^2 + H_5^{-1/3}(dx'^5)^2 + 2\xi_6 \, H_5^{-1/3} dx'^4 dx^6 + 2\xi_{10} \, H_5^{-1/3} dx'^5 dx^{10} \, . \end{aligned} \tag{3.4}$$

Let us ignore for the moment the effect of the coordinate change in the 3-form background, and take $C_3 = \omega \, dx^6 \, dx^{10}$. Its full discussion is postponed to section 3.1.4. Also, in what follows, we drop the primes of the new 45 coordinates.

Upon reduction to IIA, we obtain the background

$$\begin{aligned} e^{4\phi/3} &= H_5^{-1/3}(H_5+\xi_{10}^2) \\ A_5 &= \frac{\xi_{10}}{H_5+\xi_{10}^2} \\ ds^2_{IIA} &= H_5^{-1/2} \, (H_5+\xi_{10}^2)^{1/2} \, ds^2_{0123} + H_5^{1/2} \, (H_5+\xi_{10}^2)^{1/2} \, ds^2_{789} + \\ &\quad + H_5^{-1/2} \, (H_5+\xi_{10}^2)^{1/2} \, (H_5+\xi_6^2) \, (dx^6)^2 + 2\xi_6 \, H_5^{-1/2} \, (H_5+\xi_{10}^2)^{1/2} \, dx^4 \, dx^6 + \\ &\quad + H_5^{-1/2} \, (H_5+\xi_{10}^2)^{1/2} \, (dx^4)^2 + H_5^{1/2} \, (H_5+\xi_{10}^2)^{-1/2} \, (dx^5)^2 \end{aligned} \tag{3.5}$$

and we have the 2-form background

$$B_{NSNS} = \omega \, dx^6 \, . \tag{3.6}$$

Application of the T-duality rules in appendix A leads to the type IIB background.[1]

$$\begin{aligned} ds^2_{IIB} &= H_5^{-1/2}\,(H_5+\xi_{10}^2)^{1/2}\,ds^2_{0123} \,+\, H_5^{1/2}\,(H_5+\xi_{10}^2)^{1/2}\,(H_5+\xi_6^2)^{-1}\,(dx^4)^2 \,+ \\ &\quad + H_5^{1/2}\,(H_5+\xi_{10}^2)^{-1/2}\,(dx^5)^2 + \\ &\quad + H_5^{1/2}(H_5+\xi_{10}^2)^{-1/2}(H_5+\xi_6^2)^{-1}(dx^6+\omega_i dx^i)^2 + H_5^{1/2}(H_5+\xi_{10}^2)^{1/2}ds^2_{789} \\ e^{-\phi} &= \left(\frac{H_5+\xi_6^2}{H_5+\xi_{10}^2}\right)^{1/2} \\ B_{NSNS} &= -\,\frac{\xi_6}{H_5+\xi_6^2}\,(\,dx^6\,+\,\omega_i\,dx^i\,)\wedge dx^4 \\ B_{RR} &= -\,\frac{\xi_{10}}{H_5+\xi_{10}^2}\,(\,dx^6\,+\,\omega_i\,dx^i)\wedge dx^5\,. \end{aligned} \tag{3.7}$$

Again, more careful discussion of the M-theory 3-form (see section 3.1.4) introduces additional pieces in the above 2-forms.

As discussed above, it corresponds to a (deformed) Taub-NUT background with 3-form fluxes. Note that the flux densities are controlled by the parameters ξ_6, ξ_{10}. Their field strengths

$$\begin{aligned} H_3 &= -\xi_6\,[\,d(H_5+\xi_6^2)^{-1}\,(dx^6+\omega)\,+\,(H_5+\xi_6^2)^{-1}\,d\omega\,]\wedge dx^4 \\ F_3 &= -\xi_{10}\,[\,d(H_5+\xi_{10}^2)^{-1}\,(dx^6+\omega)\,+\,(H_5+\xi_{10}^2)^{-1}\,d\omega]\wedge dx^5 \end{aligned} \tag{3.8}$$

correspond to harmonic 2-forms in the above deformed geometry, in the following sense. As the flux parameters approach zero $\xi_6, \xi_{10} \to 0$ (and the metric approaches the undeformed Taub-NUT and the dilaton becomes constant), the field strengths are proportional to the harmonic 2-form (2.5). A sketchy description of the configuration is, therefore, a Taub-NUT geometry with 3-form fluxes (3.2). Namely, the tilting of the M5-brane with respect to the (T)duality directions in 46, and 510, turns into NSNS and RR 3-form fluxes, respectively, in the dual IIB configuration. Further backreaction of the fluxes on the geometry leads to a squashing of the latter, which appears at quadratic order in the flux perturbation.

The generalization of the duality relation is clear. Type IIB on a Taub-NUT geometry, with 3-form fluxes[2]

$$\begin{aligned} H_3 &= \Omega\,(a_1 dx^4 + a_2 dx^5) \\ F_3 &= \Omega\,(a_3 dx^4 + a_5 dx^5) \end{aligned} \tag{3.9}$$

is dual to a configuration of M5-branes with volume spanning $\mathbf{M_4}$ times the 2-plane defined by $x^6 = a_1\,x^4 + a_2\,x^5$, $x^{10} = a_3\,x^4 + a_5\,x^5$.

The generalization to arbitrary complex IIB coupling is similar, by introducing an arbitrary mixing between the coordinates 6, 10 in (3.3), via a change of variables $x^6 = x'^6 + \rho_{10}x^{10}$. The full IIB solution may be found from the above supergravity background

[1] The metric has a more symmetric form in the Einstein frame, as required by S-duality, which amounts to an exchange of the roles of ξ_6, ξ_{10}.

[2] By this we mean that these would be the expressions for the fluxes in the undeformed Taub-NUT metric. The backreaction of the flux on the metric subsequently changes the form of the flux itself. Hence in this discussion we work in the 'flux probe' approximation.

by performing an SL$(2,\mathbb{R})$ transformation. A sketchy description is that the familiar 3-form flux combination $G_3 = F_3 - \Phi H_3$ (with Φ the IIB complex coupling) plays the role of the M5-brane position in the complex plane $z = x^{10} - \Phi x^6$. For simplicity we will restrict to the above situation with zero axion in most of the paper.

Hence, we have found a duality relation that provides a geometric interpretation for 3-form fluxes in Taub-NUT geometries. In the following we discuss the geometric interpretation of diverse properties of fluxes. For simplicity, we center on fluxes of the form (3.2), rather than (3.9).

3.1.3 Flux quantization

Consider a 1 center Taub-NUT geometry and take the directions 4, 5 to be compactified on a rectangular two-torus, so that x^4, x^5 have lengths R_4, R_5. Let us introduce the 1-forms $d\phi^4 = dx^4/R_4$, $d\phi^5 = dx^5/R_5$, with period 1 over the non-trivial cycles in this two-torus. In this compact setup, consistency of the configuration requires the 3-form fluxes H_3, F_3 to be quantized. At linear order (i.e. in the 'flux probe' approximation) this can be phrased as

$$H_3 \;=\; k_6\,\Omega\,d\phi^4 \;+\; k_6'\,\Omega\,d\phi^5 \tag{3.10}$$

with $k_6, k_6' \in \mathbb{Z}$, and analogously for F_3 (with coefficients k_{10}', k_{10}). Namely the fluxes must define *integer* cohomology classes.

The quantization of fluxes has a very natural interpretation in the dual M-theory configuration. Consider for simplicity fluxes $H_3 = k_6 \Omega d\phi^4$, $F_3 = k_{10}\Omega d\phi^5$. Recalling the interpretation of the flux coefficients as dual positions (2.10), we have

$$\frac{\partial \phi^6}{\partial x^4} = \frac{k_6}{R_4}\,; \qquad \frac{\partial \phi^{10}}{\partial x^5} = \frac{k_{10}}{R_5}\,. \tag{3.11}$$

This implies that the total wrapping number of the M5-brane in the direction 6, as one winds around the direction 4, is an integer

$$\int_0^{R_4} (\partial\phi^6/\partial x^4) dx^4 = k_6 \tag{3.12}$$

and analogously for the wrapping number of the M5-brane in 10 as one moves in 5.

Integrality of the cohomology class of the 3-form fluxes is thus dual to the integrality of the homology class of the M5-brane, namely that the wrapping numbers along the cycles of the compact directions should be integers.

3.1.4 Induced charges

An interesting property of NSNS and RR 3-form fluxes is that their wedge product acts as a source for the RR 4-form. Namely, the combination of fluxes carries an induced D3-brane charge. In fact, these are not the only induced charges carried by our flux configurations. In this section we describe them and their dual geometric interpretation.

In order to have finite charges we need to consider, as above, compact 4, 5 directions, which we take of unit length for simplicity. Consider in the M-theory picture an M5-brane wrapped along one 1-cycle in the 4-6 two-torus and along one 1-cycle in the 5-10 two-torus.

The charges of the configuration are described by the homology class

$$
\begin{aligned}
[\Pi] &= (k_4[a_4] + k_6[a_6]) \otimes (k_5[a_5] + k_{10}[a_{10}]) \\
&= k_4k_5[a_4] \otimes [a_5] + k_6k_{10}[a_6] \otimes [a_{10}] + k_4k_{10}[a_4] \otimes [a_{10}] + k_6k_5[a_6] \otimes [a_5]\,, \quad (3.13)
\end{aligned}
$$

where $k_i \in \mathbb{Z}$ and $[a_i]$ is the 1-cycle along the i^{th} direction.

The charge k_4k_5 is mapped to the Taub-NUT charge (i.e. number of centers) in the type IIB dual. The charge k_6k_{10} should in principle be mapped to a D3-brane charge in the IIB dual. Indeed, this is the induced charge due to the 3-form fluxes, as follows from

$$
\begin{aligned}
N_{D3} &= \int_{TN\times\mathbf{T^2}} H_3 \wedge F_3 = \tan\theta_1 \tan\theta_2 \int_{TN} \Omega\wedge\Omega \int_{\mathbf{T^2}} dx^4\, dx^5 \\
&= k_4k_5 \tan\theta_1 \tan\theta_2 \;=\; k_6\, k_{10}\,. \qquad (3.14)
\end{aligned}
$$

Hence we have found a geometric interpretation for the 4-form charge carried by 3-form fluxes.

There remains to interpret the charges k_4k_{10} and k_6k_5. They correspond to charges of D5-branes spanning 46 and NS5-branes spanning 56, respectively. Their presence can be understood from the central charge formula for the configuration. General results (see [12] or [13, section 3.1]) imply that an N-center Taub-NUT geometry with a p-form background asymptotically along x^6 develops an induced charge of the corresponding magnetic object. In our context, a Taub-NUT charge $N = k_4k_5$ in the presence of a NSNS 2-form asymptoting to $\xi_6 dx^4 dx^6$ develops Q units of induced charge of NS5-brane along 012356, with

$$
Q = \xi_6 k_4 k_5 = \tan\theta_1 k_4 k_5 = k_6 k_5\,, \qquad (3.15)
$$

as required. Analogously for the D5-brane charge induced by the RR 2-form background.

In the following we rederive these results. Indeed, all the induced charges are correctly reproduced once we take into account the change of the M-theory 3-form background under the coordinate change (3.3), ignored in section (3.1.2) for simplicity.

After the coordinate reparametrization (3.3), the 11d background 3-form becomes

$$
C_3 = \omega\, dx^6\, dx^{10} - \xi_6\omega\, dx^4\, dx^{10} - \xi_{10}\omega\, dx^6\, dx^5 + \xi_6\xi_{10}\omega\, dx^4\, dx^5 \qquad (3.16)
$$

Upon reduction to IIA, we obtain the p-form background

$$
\begin{aligned}
B_{NSNS} &= \omega\, dx^6 - \xi_6\, \omega\, dx^4 \\
C_3 &= -\xi_{10}\, \omega\, dx^6\, dx^5 + \xi_6\, \xi_{10}\, \omega\, dx^4\, dx^5\,. \qquad (3.17)
\end{aligned}
$$

Further T-duality along the direction 6 leads to

$$
\begin{aligned}
B_{NSNS} &= -\frac{\xi_6}{H_5 + \xi_6^2}\,(\,dx^6 \,+\, \omega_i\, dx^i\,) \wedge dx^4 \,-\, \xi_6\, \omega \wedge dx^4 \\
B_{RR} &= -\frac{\xi_{10}}{H_5 + \xi_{10}^2}\,(\,dx^6 \,+\, \omega_i\, dx^i) \wedge dx^5 \,+\, \xi_{10}\, \omega \wedge dx^5 \\
C_4 &= \xi_{10} \left(\xi_6 + \frac{\xi_6}{H_5 + \xi_6^2} \right) \omega \wedge dx^4 \wedge dx^5 \wedge dx^6\,. \qquad (3.18)
\end{aligned}
$$

As we now show, the additional pieces in the p-form background contain the additional induced charges discussed above. We sketch their computation, using expressions correct to the corresponding order in the fluxes (linear for the NS5-, D5-brane charges, quadratic for the D3-brane charge). The NS5- and D5-brane charges can be obtained by integration of the NSNS resp. RR field strength 3-forms over the 3-cycles of the form $\mathbf{S^2}$ times the direction x^4 resp. x^5, where the $\mathbf{S^2}$ is a 2-sphere in the $\mathbb{R}^3$ base of Taub-NUT, surrounding the centers. This integral vanishes for the first piece of the field strengths (3.8), since they are harmonic and the 3-cycle is homologically trivial. Using (2.2), the integral of the new piece however gives

$$\int_{\mathbf{S^2}\times(\mathbf{S^1})_4} \xi_6\, d\omega\, dx^4 \;=\; k_4\, k_5\, \xi_6 \;=\; k_5\, k_6 \tag{3.19}$$

for the NSNS field, and analogously for the RR field. Hence we recover the correct charges.

The induced D3-brane charge is computed similarly. The charge arises from integrating the 5-form flux over the 45 two-torus times a large $\mathbf{S^3}$ in Taub-NUT. The latter is obtained from the $\mathbf{S^1}$ fibration over an $\mathbf{S^2}$ in the base. Recalling the modification of the 5-form field strength due to Chern-Simons couplings,

$$\tilde{F}_5 = dC_4 - \frac{1}{2}B_{RR}\wedge H_3 + \frac{1}{2}F_3 \wedge B_{NSNS} \tag{3.20}$$

we have

$$\int_{\mathbf{S^3}\times\mathbf{T^2}} \tilde{F}_5 = \int_{TN\times\mathbf{T^2}} d\tilde{F}_5 = \int_{TN\times\mathbf{T^2}} H_3\wedge F_3 \tag{3.21}$$

namely the familiar contribution to the tadpole for the 4-form. In our background, the 3-form fluxes have the structure

$$H_3 = -\xi_6\Omega\wedge dx^4 - \xi_6 d\omega\wedge dx^4\,; \qquad F_3 = -\xi_{10}\Omega\wedge dx^5 + \xi_{10}d\omega\wedge dx^5 \tag{3.22}$$

Out of the four contributions to $H_3\wedge F_3$, the piece proportional to $d\omega\wedge d\omega$ does not contribute to the integral in (3.21), and the two pieces proportional to $\Omega\wedge d\omega$ cancel each other. The induced D3-brane charge arises from the piece proportional to $\Omega\wedge\Omega$ leading to a total charge of $k_4k_5\xi_6\xi_{10} = k_6k_{10}$, as in the naive discussion (3.14). Hence our flux configuration correctly accounts for all induced charges of the system. Alternatively, the M5-brane homology class provides a simple geometric interpretation for them.

In the above configuration the homology charges, q_{ij}, coefficients of the terms $[a_i]\otimes[a_j]$ in $[\Pi]$, satisfy the quadratic constraint $q_{45}q_{610} = q_{410}q_{65}$. This follows from the factorized form of the wrapped 2-cycle, and implies that the system preserves 1/2 supersymmetry. It is natural to wonder about M5-brane configurations were the homology charges q_{ij} do *not* satisfy a quadratic constraint. As discussed in [14], they correspond to M5-branes wrapped on non-factorizable holomorphic 2-cycles, which preserve 1/4 of the supersymmetries. These configurations are easily obtained from recombination of factorizable M5-branes intersecting at angles not defining an SU(2) rotation. The IIB dual of such systems is briefly discussed in section 3.2.3.

3.1.5 Supersymmetry

Another interesting property of flux configurations is the amount of supersymmetry that they preserve. There exist in the literature several classes of supersymmetric fluxes (see e.g. [15]), but a unified understanding of them is lacking. In this section we argue that our IIB flux configurations provide a family of supersymmetric fluxes that interpolates between two classes of familiar supersymmetric configurations.[3]

Indeed, considering the case of $\xi_6 = \pm\xi_{10} \equiv \xi$, our flux background $G_3 = F_3 - \Phi H_3$ (with Φ the IIB complex coupling) satisfies the imaginary self- (or anti-self-)duality condition:

$$G_3 = \pm i *_{6d} G_3 \,. \tag{3.23}$$

In fact, the fluxes can be seen to be $(2,1)$ [or $(1,2)$] and primitive. Moreover the dilaton-axion fields are constant, and the metric takes the form

$$\begin{aligned} ds^2_{IIB} = {} & H_5^{-1/2}(H_5+\xi^2)^{1/2} ds^2_{0123} + H_5^{1/2}(H_5+\xi^2)^{-1/2} \times \\ & \times \left\{ (dx^4)^2 + (dx^5)^2 + (H_5+\xi^2) ds^2_{789} + (H_5+\xi^2)^{-1}(dx^6+\omega)^2 \right\} . \end{aligned} \tag{3.24}$$

It is a warped version of $\mathbf{M_4}$ times $\mathbb{R}^2_{(45)}$ times the Taub-NUT. This background is a particular case of the class considered in section 3 in [15] generalizing the fluxes in [16] (and which also appears in compact models, see e.g. [3]).

On the other hand, for $\xi_6 \neq \xi_{10}$, the flux G_3 does not have any particular self-duality property, the dilaton has a non-trivial profile, and the metric deviates from the warped ansatz. However, although the configuration is more involved, the M-theory picture implies that it is supersymmetric as well (preserving 16 supercharges). On one hand the initial M5-brane configuration is clearly supersymmetric, and due to the smearing, the constant spinors do not depend on the duality coordinates 610, hence are also preserved in the dual IIB image [17]. From a different point of view, the configuration satisfies the equations of motion, i.e. minimizes the energy, in a sector where the central charges satisfy a quadratic constraint, hence ensuring that the energy-minimizing state is 1/2 BPS, namely preserves 16 supersymmetries. It would be interesting to verify explicitly that the supersymmetric variations vanish in our background.

This more general class of supersymmetric fluxes, with varying dilaton, is highly reminiscent of the flux configurations appeared in [6] and studied in [15] (to first order in the fluxes). Indeed, the supersymmetries preserved correspond to the supersymmetry preserved by bound states of D3-branes and 5-branes. Moreover, as we discuss in section 3.3, such fluxes induce mass terms and dielectric polarization on D3-brane probes when the latter are introduced, in close analogy to [6]. Again, it would be interesting to perform a direct comparison with the latter reference. Some differences exist, however: our configurations preserve 16 supersymmetries, exist in Taub-NUT geometries, and are associated to $\mathcal{N}=2$ mass terms on D3-brane probes (while fluxes in [6] preserve four supercharges, are introduced in (warped) flat space, and correspond to $\mathcal{N}=1$ mass terms on D3-brane probes).

[3] After completion of this work, we noticed [29] which discusses a (potentially related) family of fluxes with interpolating supersymmetries. It would be interesting to understand the relation between both approaches.

Leaving a more detailed discussion of this interesting property of our configurations for future work, we conclude by emphasizing that our duality provides a simple construction of a family of supersymmetric flux configurations interpolating between well-known classes of supersymmetric fluxes.

3.1.6 Relation to magnetized D6-branes

Another interesting dual realization of the above systems is obtained as follows. Consider type IIB on a Taub-NUT geometry in 6789, momentarily without fluxes. Let us T-dualize along the direction 3, to obtain IIA theory on a Taub-NUT background. Now perform a '9-11 flip', namely lift the configuration to M-theory on a Taub-NUT background, and shrink the isometry direction of the latter; the resulting configuration is a type IIA D6-brane. Let us now carry out the same exercise in the presence of type IIB 3-form fluxes of the form (3.2). T-dualizing to IIA we obtain a Taub-NUT geometry with fluxes

$$H_3 = \xi_6\, \Omega\, dx^4\,; \qquad G_4 = \xi_{10}\, \Omega\, dx^3\, dx^5\,. \tag{3.25}$$

Lifting to M-theory, we obtain a Taub-NUT geometry with 4-form flux

$$G_4 = \xi_6\, \Omega\, dx^4\, dx^{10} + \xi_{10}\, \Omega\, dx^3\, dx^5\,. \tag{3.26}$$

This can be compared with fluxes in M-theory compactifications in [2]. It is easy to recover different properties of the latter from our description in terms of M5-brane geometries.

Reducing along the isometrical direction of Taub-NUT, we obtain a IIA D6-brane. Now recall that D6-brane world-volume gauge fields arise from components of the M-theory 3-form along the harmonic 2-forms, i.e. of the type $C_3 = \Omega\, A_1$. We conclude that the configuration includes D6-brane world-volume magnetic fields

$$F_2 = \xi_6\, dx^4\, dx^{10} + \xi_{10}\, dx^3\, dx^5\,. \tag{3.27}$$

Namely there are constant magnetic fields ξ_6, ξ_{10} in the 2-planes 410 and 35, respectively. The imaginary self- (or anti-self-)duality conditions on the original IIB fluxes correspond to the (anti)self-duality of the world-volume gauge background. This dual picture provide interesting complementary viewpoints on our previous discussion and our proposals below.

3.2 Multiple M5-branes and twisted fluxes on orbifolds

3.2.1 Relation to twisted fluxes in $\mathbb{C}^2/\mathbb{Z}_N$ orbifolds

There is a natural generalization of the setup in section 3.1, namely considering several M5-branes. Consider the M-theory picture, and introduce N M5-branes, labeled $a = 1, \ldots, N$, spanning 0123 and the 2-planes

$$\begin{aligned} x^6 &= \tan\theta_{1,a}\, x^4 \\ x^{10} &= \tan\theta_{2,a}\, x^5\,. \end{aligned} \tag{3.28}$$

The dual IIB configuration is given by an N-center Taub-NUT space, with 3-form fluxes along the harmonic 2-forms Ω_a, namely

$$\begin{aligned} H_3 &= \sum_{a=1}^{N} \tan\theta_{1,a}\, \Omega_a\, dx^4 \\ F_3 &= \sum_{a=1}^{N} \tan\theta_{2,a}\, \Omega_a\, dx^5\,. \end{aligned} \tag{3.29}$$

Generalizations are straightforward and will not be discussed explicitly. It is interesting to realize that we in general have fluxes in the compact non-trivial 2-cycles of the geometry. Namely, the projection of the flux along a particular harmonic 2-form Ω_{ab} is related to the relative angle between the corresponding dual M5-branes a, b. The M5-brane picture allows a reliable description of the system even in the limit of coincident M5-branes, namely when the 2-cycles are collapsed to zero size and the geometry develops an orbifold singularity [4]. The present setup, therefore allows a reliable description of untwisted and twisted fluxes at orbifold singularities $\mathbb{C}^2/\mathbb{Z}_N$. The fact that the description and properties of the fluxes are reliable even in this large curvature regimes is clearly related to the large amount of supersymmetry preserved by the configuration.

3.2.2 Moduli stabilization and supersymmetry

One of the most interesting features of configurations with fluxes is that minimization of the vacuum energy density leads to a constraint on the flux density, namely the flux combination $G_3 = F_3 - \Phi H_3$ (with Φ the IIB complex coupling) must be imaginary self-dual (or anti-self-dual)

$$G_3 = \pm i *_{6d} G_3 \tag{3.30}$$

Since, due to flux quantization, flux densities are functions of moduli, the above condition fixes their vevs. In our particular context of fluxes in Taub-NUT geometries, the above condition is moreover equivalent to the conditions that the flux G_3 is $(2,1)$ [or (1,2)] and primitive (namely $G_3 \wedge J = 0$). That is, the flux preserves a particular supersymmetry. In this section we discuss how the above condition, moduli stabilization, and supersymmetry, are encoded in the M-theory configuration.

Consider compactifying the directions 45 in a two-torus, for simplicity rectangular, with x^4, x^5 of lengths R_4, R_5. Consider a configuration with two M5-branes, which we take without loss of generality at angles $\pm(\theta_1, \theta_2)$ in the two-planes 46 and 510, respectively. For simplicity, consider them to have wrapping numbers k_6, k_{10} along 6, 10 (and wrapping once along 4, 5). We have

$$\xi_6 = \tan\theta_1 = \frac{k_6 R_6}{R_4}\,; \qquad \xi_{10} = \tan\theta_2 = \frac{k_{10} R_{10}}{R_5}\,. \tag{3.31}$$

[4]Such fluxes have appeared e.g. in $\mathcal{N}=2$ models in [25].

In the dual IIB picture we have a two-center Taub-NUT space with fluxes

$$H_3 = 2k_6\, \Omega_{12}\, d\phi^4\,; \qquad F_3 = 2k_{10}\, \Omega_{12}\, d\phi^5 \tag{3.32}$$

with $\Omega_{12} = (\Omega_1 - \Omega_2)/2$ and ϕ^4, ϕ^5 are defined as in section 3.1.3. Namely, it corresponds to a configuration with purely twisted fluxes.

Introducing the complex coordinate $z = \phi^4 + \tau\phi^5$ with $\tau = iR_5/R_4$, we have

$$G_3 \;=\; 2\Omega_{12}\,\left[\frac{dz}{\tau-\overline{\tau}}\,(\,k_{10} + \Phi\,\overline{\tau}\,k_6\,)\;+\;\frac{d\overline{z}}{\tau-\overline{\tau}}\,(\,-k_{10} - \Phi\,\tau\,k_6\,)\right]. \tag{3.33}$$

The condition that G_3 is imaginary self-dual ((2,1) and primitive) requires the last piece to drop, hence the dual of (3.30) is

$$\frac{k_{10}R_{10}}{R_5} = \frac{k_6 R_6}{R_4} \tag{3.34}$$

namely $\theta_1 = \theta_2$. This is the familiar condition[5] that the rotation relating the two M5-branes is in SU(2). This is a condition of minimization of the energy of the two M5-brane system. When regarded as function of the moduli, the condition fixes the vevs of the latter, illustrating the dual version of moduli stabilization by fluxes.

Finally, for configurations of branes intersecting at two non-trivial angles, the energy minimization conditions automatically imply the supersymmetry conditions. This is dual to the statement that in Taub-NUT spaces, (anti)self-duality of the fluxes automatically implies supersymmetry of the configuration.

3.2.3 Stabilization of non-susy flux configurations; enhancons

Branes intersecting at angles *not* in SU(2) relation suffer an instability against recombination to a single smooth curve, whose volume is smaller than the original intersecting configuration. This follows from analyzing the BPS formula for the system, or by using a dual intersecting D-brane configuration. It is interesting to consider the interpretation of this process in the IIB dual realization of this system. In this section we discuss this interpretation in the case of none of the branes being along the directions 610.[6] A useful approach to this instability is to consider the supersymmetric case first, where the recombination process is parametrized by a modulus. Let $z = x^4 + ix^5$, $w = x^6 + ix^{10}$. The intersecting configuration is described by an M5-brane wrapped on a curve of the form $(z-w)(z+w) = 0$, while in the recombined configuration the brane wraps the cycle $(z-w)(z+w) = \epsilon$.

Consider as in section 3.2.2 two M5-branes at angles $\pm(\theta_1, \theta_2)$ in the 46 and 510 directions, respectively, with wrapping numbers $(1, k_6)$ and $(1, k_{10})$, and which coincide in 789. The dual configuration contains a two-center Taub-NUT geometry, with coincident

[5] The possibility $\theta_1 = -\theta_2$ is dual to the (also energy minimizing) situation of G_3 being imaginary anti-self-dual fluxes (which in the Taub-NUT geometry is equivalent to being $(1,2)$ and primitive).

[6] This situation is considered in section 3.3.2, and has a somewhat different dual interpretation, in terms of D3-brane polarization.

center, and with 3-form twisted fluxes

$$H_3 = 2\,\xi_6\,\Omega_{12}\,dx^4\,; \qquad F_3 = 2\,\xi_{10}\,\Omega_{12}\,dx^5\,. \tag{3.35}$$

For $\xi_6 \neq \xi_{10}$, the flux configuration is non-supersymmetric, and the M-theory picture suggests that it suffers an instability.

The process involves somewhat exotic degrees of freedom. In the supersymmetric case, the degrees of freedom responsible for the restabilization of the configuration are BPS states, and so can be translated directly from dual configurations of intersecting M5- or D-branes. It is easy to realize that they correspond to tensionless degrees of freedom from D3-branes wrapped on the collapsed 2-cycle Σ_{12}. D3-branes wrapped on the Σ_{12} lead to tensionless objects because there are points in 45 where the integral of the 2-form fields over Σ_{12} vanish. The number of such points, and hence of the massless degrees of freedom involved in the condensation, is given by $4k_6k_{10}$, in agreement with the number of intersection points in the dual.

The nature of the configuration after condensation of these degrees of freedom is unclear, since it involves a Higgs effect for the 2-form fields. It is tantalizing to propose that its description in the IIB configuration is related to the enhancon geometries [18], since it involves a shell region (dual to the region of recombination of the dual M5-branes) associated to massless D-brane degrees of freedom. Indeed, in M-theory the final configuration is an M5-brane wrapped on a single smooth curve. The IIB version of this kind of configuration in theories with 8 supercharges has been discussed in [19]. It would be very interesting to develop these relations further.

Although the decay involves a configuration not very familiar in the context of fluxes, it is perfectly natural in other dual realizations of the system. Consider for instance the dual configuration in section 3.1.6, of D6-branes with world-volume magnetic fluxes. In the non-supersymmetric case the abelian gauge background decays to an $U(2)$ instanton involving the non-abelian degrees of freedom, arising from open strings stretching between the two branes. The non-abelian character of the final configuration is thus responsible for the difficulty in finding a supergravity description on the IIB side.

Note that the above argument provides and answer to the question raised in section 3.1.4 on the IIB dual to M5-branes wrapped on non-factorized holomorphic cycles. Interestingly, the latter provide a geometric picture of a somewhat mysterious process in IIB theory.

3.2.4 Stabilization/supersymmetrization of non-susy geometries

There is an interesting dual realization of this kind of configuration, in which one of the angle parameters maps to a flux background, while the second angle leads to a non-Calabi-Yau geometry in the dual. The full configuration is however stable and supersymmetric, due to a compensating effect of the flux and the geometry.

Consider two M5 branes spanning 0123, at angles $\pm(\theta_1, \theta_2)$ in the 2-planes 46 and 510, and located at points in 789. Consider the coordinate 9 to be compact, and consider reducing to a type IIA configuration by shrinking it. We obtain two IIA NS5-branes spanning 0123, at angles θ_1 in 46 and θ_2 in 510. Let us now perform a T-duality along the direction 10. In the case of $\theta_2 = 0$, the T-dual configuration is a purely geometric background that corresponds to a non-supersymmetric and unstable background, described by a 5d geometry $\mathbf{X_5}$, considered in [20]. Intuitively, it corresponds to two Taub-NUTs intersecting in a non-supersymmetric fashion. The configuration contains an instability localized at the singular point at the intersection. The relaxation of the instability was shown to correspond to a dynamical resolution of the singularity, leading to a smooth geometry with a non-trivial 2-sphere, which in the non-compact case runs away to infinite size.

For non-zero θ_2, the new non-trivial angle does not modify the topology dual geometry. The IIB configuration is given by a squashed version of the same topological manifold $\mathbf{X_5}$, which in addition now contains a non-zero NSNS 3-form flux.

For the case $\theta_1 = \pm\theta_2$, namely for a 3-form flux tuned to the 5d geometry, the configuration is supersymmetric. In this sense, the introduction of the 3-form flux stabilizes the geometry $\mathbf{X_5}$. In fact, the unstable direction, which corresponds to increasing the size of the non-trivial 2-sphere, is a modulus direction (dual to the recombination of the two intersecting branes in the susy case). It is interesting to notice that in the large volume limit of this modulus, supergravity is reliable and the stabilization mechanism is amenable to explicit analysis.

Clearly $SL(2,\mathbb{R})$ transformations of the above configuration (equivalently, shrinking the directions 910 in different fashions) show that the stabilization can be achieved for a suitably tuned amount of RR flux or a combination of NSNS and RR fluxes.

Although the above non-supersymmetric geometries are not of orbifold kind, it is tempting to speculate that flux configurations in non-supersymmetric orbifold may also lead to the stabilization of such geometries (although perhaps not in a supersymmetric fashion). This presumably arises from the competition of the tachyonic nature of twisted sector scalars and the contribution to their potential generated by a background of twisted fluxes. It would be interesting to analyze this idea in more detail.

3.2.5 Dualities to conifolds

Finally, we would like to point out that the above configurations admit a dual version where both angles become properties of the geometry; indeed, the configurations are dual to a purely geometric background containing a conifold singularity. Consider two M5-branes intersecting at SU(2) angles,[7] and consider compactifying, and subsequently shrinking, the directions 78. For instance, shrinking along 8, we obtain a configuration of intersecting IIA NS5-branes; and T-dualizing along 7, we obtain [21] IIB theory on a singular geometric background with a conifold singularity.

Notice that the two dualities are related in the M-theory picture by an exchange of the directions 67 and 810, namely a simultaneous $SL(2,\mathbb{Z})$ transformation in the corresponding two two-tori. In the IIB picture, the configuration of Taub-NUT with fluxes and the conifold

[7] The duality for non-supersymmetric angles can be performed similarly, and leads to non-supersymmetric geometries of conifold type, considered in [20].

geometry are thus related by a U-duality transformation. It is clear that one can generate even more duals by choosing to dualize along other intermediate directions in the two-tori 67 and 810. All these would fill out a multiplet of highly non-trivial configurations under U-duality.

3.3 D-brane probes

In this section we consider several properties of the physics of D3-brane probes in the above Taub-NUT plus flux configurations, gaining insight from the dual M-theory realization. In the latter, the D3-branes correspond to M5-branes along 0123610.

3.3.1 Mass deformations

The M-theory configurations we are considering have an interesting application. Configurations of M5-branes (reducing to NS5- and D4-branes in IIA) have been considered in [22] to study the dynamics of the $\mathcal{N}=2$ 4d supersymmetric gauge theories on the non-compact part of their world-volume. In particular, the situation with one M5-brane along 012345 and N M5-branes along 0123610 leads to an $\mathcal{N}=2$ $U(N)$ gauge theory with an adjoint hypermultiplet (that is, $\mathcal{N}=4$ super-Yang-Mills).

An interesting deformation of this theory corresponds to the introduction of a complex mass parameter for the hypermultiplet. In the brane realization, it was argued in [22] to correspond to embedding the M5-brane configuration in a non-trivial geometry in 45610, defined by a twisted identification (for diagonal metric in 610)

$$x^6 \to x^6 + 2\pi R_6 \, ; \qquad v = x^4 + ix^5 \to v + m \, . \tag{3.36}$$

In the IIA picture, when D4-branes along 01236 are introduced, the shift implies that it is energetically favourable for them to break at the NS5-brane location, see figure 3 *a*, *b*. The separation m between the D4-brane is translated into a hypermultiplet mass. In the M-theory picture, the breaking corresponds to a recombination of the different kinds of M5-branes, figure 3c, which due to the twisting no longer intersect in SU(2) angles. The holomorphic curve wrapped by the recombined M5-brane after recombination is the Seiberg-Witten curve of the mass-deformed theory.

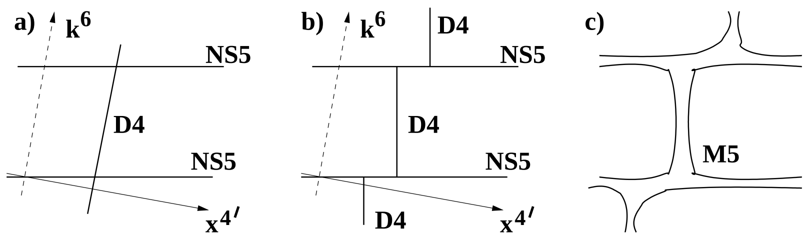

Figure 3: The type IIA picture dual to IIB D3-branes in the presence of flux corresponds to NS5/D4 brane configurations in a twisted $\mathbf{S^1} \times \mathbb{R}$. *a*) The duals to D3-branes (wrapped D4-branes) are attracted (in 789) towards the duals of the Taub-NUT center (NS5-branes). *b*) Once there, they break on the NS5-branes and relax to a supersymmetric configuration; in the T-dual this corresponds to a dielectric polarization. *c*) The M-theory picture of *b*) is a recombination of initially intersecting M5-branes.

It is easy to realize that the configuration of the M5-brane along 012345 in the twisted geometry (3.36) is one particular case of our M-theory configurations. Indeed, consider one M5-brane along 012345, and introduce the variables

$$x^4 = x'^4 - \frac{\mathrm{Re}(m)}{2\pi R_6} x^6 \, ; \qquad x^5 = x'^5 - \frac{\mathrm{Im}(m)}{2\pi R_6} x^6 \, . \tag{3.37}$$

We see that at fixed values of the coordinates x^4, x^5, there is an induced shift of x'^4, x'^5 when x^6 is shifted, precisely given by (3.36). Hence, we reach the conclusion that our configurations provide a realization of the mass deformation of $\mathcal{N} = 4$ super-Yang-Mills, once suitable probes are introduced. It is useful to see how this arises in the dual IIB picture. We have a configuration of N D3-branes in a Taub-NUT geometry with H_3 flux roughly of the form $H_3 = \Omega \wedge (m_4 dx^4 + m_5 dx^5)$. The configuration does not preserve supersymmetry, and the D3-branes suffer an attractive force that pins them to the Taub-NUT center. The pinning potential for the adjoint hypermultiplet, has been discussed in a dual realization in [13]. The mass term is as expected given by m.

A last tricky point is restoration of supersymmetry, which is provided by the T-dual process of the D4-brane breaking on the NS5-brane. This effect will be considered in more detail in next section, where it is shown to correspond to a dielectric polarization of the D3-branes, as in [7]. Indeed this is suggested by the fact that the two endpoints of the D4-brane are oppositely charged under the NS5-brane world-volume fields, and they are separated (in a dipole-like fashion) due to the twisted geometry.

3.3.2 Myers dielectric effect

In this section we consider the main dynamical process taking place in certain non-supersymmetric configurations. Consider one M5-brane at angles θ_1, θ_2 in the 2-planes 46 and 510, generically with $\theta_1 \neq \pm\theta_2$. Let us now introduce M5-branes spanning 0123610; these do not preserve the same supersymmetry as the original ones. The configuration suffers from an instability against recombination of the M5-branes. Once they recombine, we end up with a single M5-brane wrapping a supersymmetric cycle, preserving a supersymmetry different from any of the original ones.

In the dual IIB configuration, we have a Taub-NUT geometry with 3-form flux G_3, generically not satisfying any (anti)self-duality condition. In this background we introduce a set of D3-brane probes. In general, they do not preserve the same supersymmetry as the background fluxes. The M-theory picture suggests there is an instability in this situation, and a mechanism that allows to restabilize the configuration and restore the supersymmetry. Indeed, there is such a process, as is suggested by the dielectric effect [7]. The world-volume action for a D3-brane in a non imaginary self-dual flux background develops trilinear couplings for the world-volume scalars ϕ_m

$$(G_3 - i *_{6d} G_3)_{lmn} \, \phi_l \, \phi_m \, \phi_n \tag{3.38}$$

which trigger polarization of the D3-brane into a fuzzy sphere [7]. It is non-trivial to compute the final endpoint of the relaxation process, including the backreaction of the dielectric D3-brane on the background. Happily the M-theory picture provides the answer and ensures that the final state is supersymmetric.

Notice the close analogy of the above discussion with the picture in [6]. Namely, a set of D3-branes immersed in a supersymmetric but non-imaginary-selfdual flux is polarized to a final configuration (including the backreaction) which preserves as a whole some supersymmetry. Moreover, in [6] the fluxes correspond to a deformation of the D3-brane world-volume theory by mass terms for the adjoint matter, precisely as in our case, as discussed in previous section. The main difference between both configurations is that in [6] the configurations preserve four supercharges, and correspond to mass terms for all $\mathcal{N} = 1$ chiral multiplets, while in our case the configuration preserves eight supercharges, and the fluxes correspond to an adjoint hypermultiplet mass.[8] It would be very interesting to compare our backgrounds with the AdS/CFT duals proposed for $\mathcal{N} = 2$ mass deformed theories in e.g. [23].

4. Conifolds and $N = 1$ fluxes

It is natural to wonder if the above duality between fluxes and brane configurations admits a generalization to background geometries different from Taub-NUT, for instance the interesting case of Calabi-Yau threefolds. Clearly, our duality has been useful because the background geometry could be dualized into a brane, which subsequently accounts for the background fluxes by a deformation of its geometry. In order to be able to repeat the exercise for some Calabi-Yau threefold, we need the corresponding geometry to be related to configurations of branes as well. Namely, we should look for an underlying duality between M5-branes and threefold geometries, analogous to that in section 2.

A natural candidate is the duality between a configuration of two M5-branes along 012345 and 012389, respectively, and type IIB theory on a threefold geometry $\mathbf{X_6}$ containing a conifold singularity [21]. An intuitive derivation is as follows. Let us start in the M-theory picture with 6, 10 compact, and M5-branes along 012345 and 012389, and let us find the dual IIB picture when we shrink 6, 10. For instance, shrinking 10 first we get a IIA configuration of two NS5-branes along 012345 and 012389. Denoting z, w the complex coordinates in the two 2-planes 45 and 89, we have an NS5-brane wrapped on the curve $zw = 0$. Let us now perform a T-duality along the direction 6. The T-dual configuration must be a purely geometric background given by 'two intersecting Taub-NUTs', namely an $\mathbf{S^1}$ fibration (with fiber parametrized by the dual x^6), degenerating over the curved wrapped by the original NS5, i.e. over the locus $zw = 0$. The fibration can be described by the equation

$$xy = zw\,, \tag{4.1}$$

where the two complex variables x, y, subject to the above constraint, encode the coordinates 67. Namely, for generic z, w, the variables x, y parametrize a $\mathbb{C}^*$, that is a cylinder; at the locus $zw = 0$ the circle degenerates to zero size, as desired. The above equation defines a conifold singularity, in the complex sense. The precise metric arising from T-duality would differ from the standard conifold metric in that our geometry has constant

[8] A further difference is that in our configuration the background geometry is Taub-NUT, rather than flat space. Hence a full comparison would require taking a decompactification limit, where Taub-NUT reduces to flat space.

asymptotic radius for the fiber, while the standard conifold is a conical singularity where all directions grow to infinity with the radius. However, both metrics agree near the origin; in fact the duality has been worked out using the (suitably smeared) M5-brane supergravity solution and the T-duality rules, showing that the near throat region of the M5-branes dualizes into a standard conifold singularity (see second reference in [21]). We will not enter into the detailed analysis.

There are two branches in the moduli space of the M5-brane configuration: the two M5-branes can be separated in the directions 67, or if they coincide in 67 they can be recombined (by an amount fixed by a complex modulus ϵ) into a single smooth M5 brane, wrapped on the curve $zw = \epsilon$. These two branches can be intuitively T-dualized into the small resolution of the conifold (with a homologically non-trivial $\mathbf{S^2}$, with size and B-field fixed by the interbrane distance in 76), and into the deformation of the conifold (with a homologically non-trivial $\mathbf{S^3}$, with size fixed by ϵ), described by a complex manifold $xy - zw = \epsilon$.

There is a notable example of configuration of fluxes in IIB theory on the (deformed) conifold which has been studied in the literature, namely the Klebanov-Strassler solution [16], proposed as a supergravity dual of the theory of fractional D3-branes on the conifold. In the remainder of this section we would like to argue that our duality relates this background with a configuration of M5-branes wrapped on a holomorphic curve in flat space. The latter is the M-theory lift of a IIA configuration with NS5-branes along 012345 and 012389, with D4-brane suspended among them. In fact, such a duality was already noticed in [16] and studied e.g. in [26]. In this section we review those results, pointing out that they can be understood in a broader context as a generalization of our previous duality for Taub-NUT spaces.

The curve represents two M5-branes along 012345 and 012389 recombined together via a piece of M5-brane along 0123610. The curve has been determined in [27] (for the case of non-compact 6, which is enough for qualitative purposes), and is given by

$$z = \epsilon w^{-1}\,; \qquad t = w^N\,, \tag{4.2}$$

where z and w parametrize 45 and 89, t parametrizes 6, 10, and ϵ is related to the amount of recombination of the branes. Its holomorphy is a reflection of the supersymmetry of the system.

In the present setup, we see that the proposed duality satisfies several points in analogy with the previous Taub-NUT examples. For instance, in the M5-brane configuration, the coordinates z, w satisfy $zw = \epsilon$. Hence the dual IIB background geometry should correspond with a deformed conifold, exactly as in [16].[9]

The fact that the M5-brane does not sit at fixed positions in t (i.e. in 610) implies the dual IIB configuration contains non-trivial F_3 and H_3 fluxes. In principle these fluxes would be turned on along harmonic forms associated to the two intersecting Taub-NUT spaces implicit in the conifold geometry. However, by taking linear combinations, the fluxes can

[9]Moreover, in both pictures the appearance of the scale ϵ is associated to strong dynamics in the low-energy gauge field theory.

be seen be turned along the non-trivial $\mathbf{S^3}$ and its dual. Using $\mathrm{SL}(2,\mathbb{Z})$ transformations, the flux over $\mathbf{S^3}$ can be chosen to be H_3, while (as we argue shortly) the flux on its dual turns out to be F_3.

Holomorphy of the curve implies that t (or 610) depends holomorphically on the coordinates of the base z, w. As we know this implies that dual flux configuration is supersymmetric, namely G_3 is $(2,1)$ and primitive, precisely as in [16]. Since this implies imaginary self-duality of the flux, it implies that fluxes along the $\mathbf{S^3}$ and its dual can be chosen to be H_3, F_3.

Finally, the amount of M5-brane spanning in 610 is mapped in the dual to the integral of $F_3 \wedge H_3$ in the dual IIB configuration. Namely, similarly to our Taub-NUT examples, the dual background reproduces the 4-form charge purely in terms of fluxes, as in [16].

The present duality admits clear generalizations to other threefolds having a simple dual in terms of M5-brane configurations. Several examples of this kind have been considered in the first reference in [21], for instance threefolds given by equations $xy = z^N w^M$ are dual to configurations of N and M M5-branes along 012345 and 012389. Unfortunately the metrics for these manifolds are not known, even in the near singularity region. We hope that a more quantitative understanding of our duality in the threefold case can lead to a more detailed understanding of fluxes in these geometries, hopefully providing new examples useful e.g. for the construction of supergravity duals of gauge theories.

5. Conclusions

In this paper we have exploited duality to relate backgrounds of 3-form field strength fluxes in IIB string theory on Taub-NUT space with configurations of M-theory M5-branes. Properties of the latter are familiar, yet can be related to interesting old and new properties of the flux configurations. Among the old properties, we have discussed flux quantization, supersymmetry conditions and moduli stabilization for fluxes, from the dual M5-brane geometry. We have also uncovered new aspects, like the possibility of stabilizing non-supersymmetric unstable compactifications by combining them with fluxes. Among the new properties, we have provided dual pictures for flux configurations in complicated (large curvature) regimes, like twisted fluxes on $\mathbb{C}^2/\mathbb{Z}_N$ orbifolds, and for complicated dynamical processes in the flux configurations, like restabilization of non-supersymmetric fluxes, and Myers dielectric effect.

These results indicate that duality properties of configurations with fluxes can be extremely useful in improving their understanding. A related direction, initiated in the study of mirror symmetry with fluxes [24], would be to understand duality properties of fluxes in compact manifolds. Indeed, we hope much active research along these lines.

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A. Buscher's rules for T-duality

The relations between background fields between IIA and IIB configurations related by T-duality along the direction y are given by (see e.g. [28]).

$$\begin{aligned}
\tilde{G}_{yy} &= \frac{1}{G_{yy}} & e^{2\tilde{\phi}} &= \frac{e^{2\phi}}{G_{yy}} \\
\tilde{G}_{\mu\nu} &= G_{\mu\nu} - \frac{G_{\mu y}G_{\nu y} - B_{\mu y}B_{\nu y}}{G_{yy}} & \tilde{G}_{\mu y} &= \frac{B_{\mu y}}{G_{yy}} \\
\tilde{B}_{\mu\nu} &= B_{\mu\nu} - \frac{B_{\mu y}G_{\nu y} - G_{\mu y}B_{\nu y}}{G_{yy}} & \tilde{B}_{\mu y} &= \frac{G_{\mu y}}{G_{yy}}
\end{aligned} \tag{A.1}$$

$$\begin{aligned}
\tilde{C}^{(n)}_{\mu\cdots\nu\alpha y} &= C^{(n-1)}_{\mu\cdots\nu\alpha} - (n-1)\frac{C^{(n-1)}_{[\mu\cdots\nu|y}G_{|\alpha]y}}{G_{yy}} \\
\tilde{C}^{(n)}_{\mu\cdots\nu\alpha\beta} &= C^{(n+1)}_{\mu\cdots\nu\alpha\beta y} + nC^{(n-1)}_{[\mu\cdots\nu\alpha}B_{\beta]y} + n(n-1)\frac{C^{(n-1)}_{[\mu\cdots\nu|y}B_{|\alpha|y}G_{|\beta]y}}{G_{yy}}\,.
\end{aligned} \tag{A.2}$$

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